Rao IIT Academy
Symbol of Excellence and Perfection
JEE | MEDICAL-UG | BOARDS | KVPY | NTSE | OLYMPIADS

Target Eng-2018

Date: 20 May 2018

Rao IIT Academy
JEE - ADVANCED
EXAM - 2018 / PAPER - 2
Solutions

PHYSICS

1.

Difficulty : Medium

Topics :

Work ,Energy and power,

$$\frac{dK}{dt} = rt$$

$$\int_0^K dK = r\int_0^t tdt$$

$$K = \frac{rt^2}{2} = \frac{1}{2}mV^2 \Rightarrow V = ct$$

Speed is proportional to time

$$\therefore a = \text{Constant}$$

hence force is constant

$$S = \frac{1}{2}at^2$$

Force is constant, hence conservative

$\therefore$ Correct options are (ABD)

2.

Difficulty : Easy

Topics :

Viscosity newton's law of viscosity,

$$|F| = NA\frac{dV}{dX}$$

$$|F| = NA\frac{\mu_0}{h}$$

$$\left|\frac{F}{A}\right| = N\frac{\mu_0}{h}$$

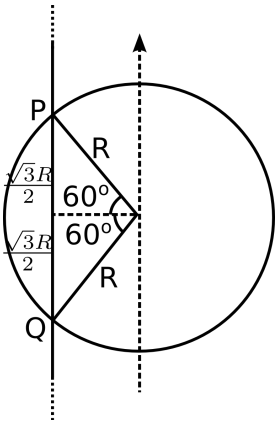
hence correct options (ACD)

3.

Difficulty : Medium

Topics :

Electrostatics,



$$\therefore \text{Charge total} = \lambda\sqrt{3}R$$

$$\therefore \text{Flux} = \frac{\rho}{\epsilon_0} = \frac{\lambda\sqrt{3}R}{\epsilon_0}$$

$\text{Electric field} = \dfrac{2K\lambda\ (\text{Normal})}{r}$ and its component along z-axis will be zero

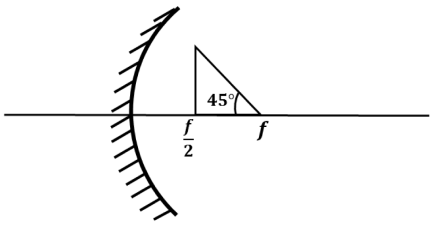
hence correct options (AB).

4.

Difficulty : Difficult

Topics :

Ray Optics,



$$m = \frac{f}{f-u}$$

$$h_I = h_0 m_T$$
$$= h_0 \frac{f}{f-x}$$
$$h_I = \tan 45 f$$
$$h_I = f$$

So option (D) is correct.

5.

Difficulty : Easy

Topics :

Modern Physics,

$$^{232}_{90}Th \rightarrow {}^{212}_{82}Pb + N_\alpha \alpha + N_\beta \beta^-$$
$$4N_\alpha = 20$$
$$\therefore N_\alpha = 5$$
$$2N_\alpha - N_\beta = 8$$
$$2 \times 5 - N_\beta = 8$$
$$N_\beta = 2$$

Hence correct options (A) and (C)

6.

Difficulty : Difficult

Topics :

Sound wave,

$$\frac{n\lambda}{2}+\frac{\lambda}{4}=50.7+e$$

$$(n+1)\frac{\lambda}{2}+\frac{\lambda}{4}=83.9+e$$

subtracting two equations

$$\therefore \frac{\lambda}{2}=33.2$$

$$\therefore \lambda=66.4cm$$

$$\therefore V=f\lambda=500\times\frac{66.4}{100}=332.0\ m/s$$

$\therefore$ Correct option (A) and (C)

7.

Difficulty : Medium

Topics :

System of particles,

$$v=v_0e^{\frac{-t}{4}}$$

$$t=0 \quad v=v_0$$

$$\therefore v_0=\frac{J}{m}=\frac{1}{0.4}=\frac{10}{4}$$

$$v_0=\frac{5}{2}$$

$$\int_0^x dx=\frac{5}{2}\int_0^{t=4}e^{\frac{-t}{4}}\,dt$$

$$x=\frac{5}{2}\times 4\left[e^{\frac{-t}{4}}\right]_0^4=10(1-e^{-1})=0.63\times 10$$

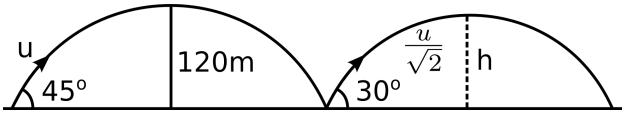
$$x=6.3m$$

8.

Difficulty : Medium

Topics :

Kinematics,



$$120 = \frac{u^2 \sin^2 45^o}{2g} \qquad h = \frac{u^2 \sin^2 30^o}{2 \quad 2g}$$

$$240 = \frac{u^2}{2g} \qquad h = \frac{240}{2} \times \frac{1}{4}$$

$$h = 30m$$

9.

Difficulty : Difficult

Topics :

Kinematics,

$$a = \frac{aE_0 \sin \omega t}{m} = \frac{1 \times 1}{10^{-3}} \sin \omega t$$

$$a = 1000 \sin \omega t$$

$$\int_0^V dV = 1000 \int_0^{\pi/\omega} \sin \omega t \; dt$$

$$V = \frac{1000}{\omega} [\cos \omega t]_{\pi/\omega}^0 = \frac{2000}{\omega} = \frac{2000}{1000}$$

$$V_{max} = 2 \; m/s$$

10.

Difficulty : Medium

Topics :

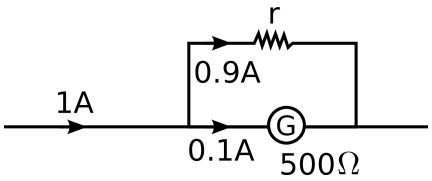
Current electricity,

$$NBAI = C\theta$$

$$50 \times \frac{2}{100} \times 2 \times 10^{-4} I = 10^{-4} \times \frac{2}{10}$$

$$10I = 1$$

$$I_g = 0.1A$$



$$0.9r = 0.1 \times 50$$

$$r = \frac{50}{9} = 5.55$$

$\boxed{r = 5.56\Omega}$ round off two decimal

11.

Difficulty : Medium

Topics :

Experiments based on using verniercalipers and screw gauge (micrometer),

$$X = \frac{FL}{AY} = \frac{12 \times 1 \times 4}{\pi \left(\frac{0.5}{1000}\right)^2 \times 2 \times 10^{11}}$$

$$X = \frac{12 \times 4 \times 10^8}{3.2 \times 25 \times 2 \times 10^{11}} m = \frac{12 \times 4 \times 4 \times 10^8}{3.2 \times 100 \times 2 \times 10^{11}} m$$

$$X = \frac{12 \times 4 \times 4}{3.2 \times 100 \times 2} mm = \frac{12 \times 4 \times 4}{32 \times 2 \times 10} mm = \frac{3}{10} mm = 0.3mm$$

$$9MSD = 10VSD$$

$$0.9 = VSD$$

$$\therefore LC = MSD - VSD$$

$$LC = 1 - 0.9$$

$$LC = 0.1mm$$

$$\therefore \text{Reading} = MSD + VSR \times LC$$

$$0.3 = 0 + VSR \times 0.1$$

$$VSR = 3$$

12.

Difficulty : Easy

Topics :

Thermodynamics,

$$T\,(8V)^{\frac{5}{3}-1} = 100(V)^{\frac{5}{3}-1}$$

$$T = \frac{100}{4} = 25$$

$$\therefore \Delta U = nC_V\Delta T = 1 \times \frac{3}{2} \times 8 \times 75$$

$$\Delta U = 900J$$

13.

Difficulty : Medium

Topics :

Modern Physics,
Photoelectric effect,

$$P = n \text{ Energy of photon}$$

$$200 = n \times 6.25 \times 1.6 \times 10^{-19}$$

$$\therefore n = \frac{200}{6.25 \times 1.6 \times 10^{-19}}$$

$$F = nmV = n\sqrt{2mKE}$$

$$F = \frac{200}{6.25 \times 1.6 \times 10^{-19}} \sqrt{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19} \times 500}$$

$$F = \frac{200}{6.25 \times 1.6 \times 10^{-19}} \sqrt{9 \times 10^{-50} \times 1.6 \times 1000}$$

$$F = \frac{200 \times 3 \times 4 \times 10^{-24}}{6.25 \times 1.6 \times 10^{-19}}$$

$$F = \frac{20 \times 3 \times 4 \times 10^{-4}}{6.25 \times 1.6}$$

$$= 6 \times 4 \times 10^{-4}$$

$$= 24 \times 10^{-4}$$

$$\therefore N = 24$$

14.

Difficulty : Easy

Topics :

Bohr's model of an atom,

$$Z^2\ 13.6\left[\left(\frac{1}{1^2}-\frac{1}{2^2}\right)-\left(\frac{1}{2^2}-\frac{1}{3^2}\right)\right]=74.8$$

$$Z^2\ 13.6\left[1-\frac{1}{4}-\frac{1}{4}+\frac{1}{9}\right]=74.8$$

$$Z^2\ 13.6\left[\frac{1}{2}+\frac{1}{9}\right]=74.8$$

$$Z^2\ 13.6\left(\frac{11}{18}\right)=74.8$$

$$Z=3$$

15.

Difficulty : Easy

Topics :

Electrostatics,

(1) Point charge at origin $E=\frac{KQ}{d^2}$

(2) $E_{\text{axial}}=\frac{2KP}{d^3}$

(3) Uniform line charge $E=\frac{2K\lambda}{d}$

(4) $E_{net}=\frac{2K\lambda}{d-L}-\frac{2K\lambda}{d+L}=\frac{2K\lambda(d+L-d+L)}{d^2-L^2}$

$$E_{net}\approx\frac{2K\lambda}{d^2}$$

(5) Infinite plane charge $E=\frac{\sigma}{2\varepsilon_0}$ constant

16.

Difficulty : Medium

Topics :

Gravitation,

$S-1$ $\qquad$ $S-2$

$v_1=\sqrt{\frac{Gm}{R_1}}$ $\qquad$ $v_2=\sqrt{\frac{Gm}{R_2}}$

$L_1=m_1v_1R_1$ $\qquad$ $L_2=m_2v_2R_2$

$K_1=\frac{GMm_1}{2R_1}$ $\qquad$ $K_3=\frac{GMm_2}{2R_2}$

$T_1=CR_1^{\frac{3}{2}}$ $\qquad$ $T_2=CR_2^{\frac{3}{2}}$

$$\frac{v_1}{v_2}=\sqrt{\frac{m_1}{m_2}}=2$$

$$\frac{L_1}{L_2}=\frac{m_1}{m_3}\ \frac{v_1}{v_2}\ \frac{R_1}{R_2}=2\times2\times\frac{1}{4}=1$$

$$\frac{K_1}{K_2}=\frac{m_1}{m_2\frac{R_1}{R_2}}=\frac{2}{\frac{1}{4}}=8$$

$$\frac{T_1}{T_2}=\left(\frac{R_1}{R_2}\right)^{\frac{3}{2}}=\left(\frac{1}{4}\right)^{\frac{3}{2}}=\frac{1}{8}$$

17.

Difficulty : Medium

Topics :

Thermodynamics,

Process (1) is adiabatic

Process (2) is isobaric $\rightarrow w=3P_0(3V_0-2V_0)=6P_0V_0$

Process (3) is isochoric $\rightarrow w=0$

Process (4) is isothermal

18.

Difficulty : Difficult

Topics :

Rotational Mechanics,

(P) $\vec{r} = \alpha t\hat{i} + \beta t\hat{j}$

$\vec{v} = \alpha\hat{i} + \beta\hat{j}$

$\vec{a} = 0\hat{i} + 0\hat{j}$

hence all conserved

$P \to 1, 2, 3, 4, 5$

(S) $\vec{r} = \alpha t\hat{i} + \frac{\beta}{2}t^2\hat{j}$

$\vec{v} = \alpha\hat{i} + \beta t\hat{j}$

$\vec{a} = 0\hat{i} + \beta\hat{j}$

L not conserved

E is only conserved

$S \to 5$

(R) $\vec{r} = a\left[\cos\omega t\hat{i} + \sin\omega t\hat{j}\right]$

$\vec{v} = a\omega\left[-\sin\omega t\hat{i} + \cos\omega t\hat{j}\right]$

$\vec{a} = -\omega^2 a\left[\cos\omega t\hat{i} + \sin\omega t\hat{j}\right]$

UCM

L, K, U, E conserved

$R \to 2, 3, 4, 5$

(Q) $\vec{r} = a\cos\omega t\hat{i} + \beta\sin\omega t\hat{j}$

$\vec{v} = -a\omega\sin\omega t\hat{i} + \beta\omega\cos\omega t\hat{j}$

$\vec{a} = -\omega^2 a\cos\omega t\hat{i} - \omega^2\beta\sin\omega t\hat{j}$

$\vec{a} = -\omega^2\left[a\cos\omega t\hat{i} + \beta\sin\omega t\hat{j}\right]$

$\vec{a} = -\omega^2\vec{r}$

L, E conserved

$Q \to 2, 5$

hnce option (A)

CHEMISTRY

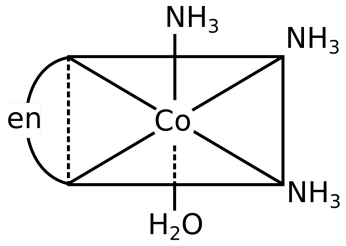
1.

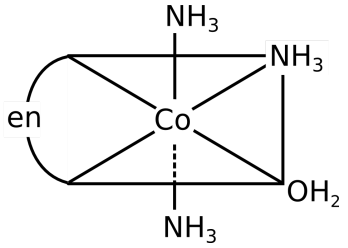
Difficulty : Medium

Topics :

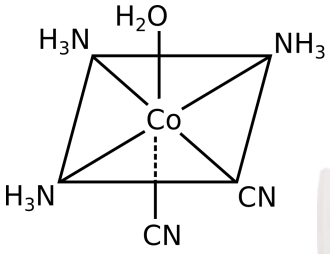
Coordination Compounds,

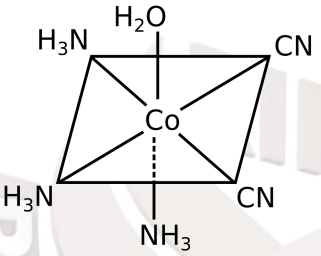
(A) $[Co(en)(NH_3)_3(H_2O)]^{3+}$ has two geometrical isomers.

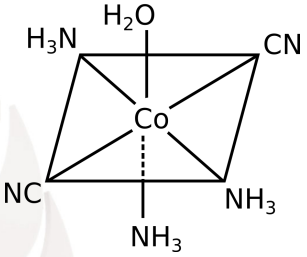




(B) If bidentate $'en'$ is replaced by two cyanide ligands.

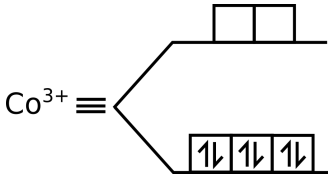






(C) $Co^{3+} = [Ar]3d^6$

In presence of SFL



Therefore it is diamagnetic in nature.

(D) H_2O is weaker ligand than NH_3, so $[Co(en)(NH_3)_3H_2O]^{3+}$ absorbs light at longer wavelength as compared to $[Co(en)(NH_3)_4]^{3+}$.

2.

Difficulty : Difficult

Topics :

Qualitative analysis,

(A) Cu^{2+} and Mn^{2+} both show characteristic green colour in the flame test, so can't be distinguished.

(B) Only Cu^{2+} can give precipitate of CuS when passing H_2S in acidic medium.

(C) Cu^{2+} and Mn^{2+} both show precipitate by passing H_2S in faintly basic medium.

(D) From electrochemical series $E^{\circ}_{Cu^{2+}/Cu} = +0.34V(SRP)$

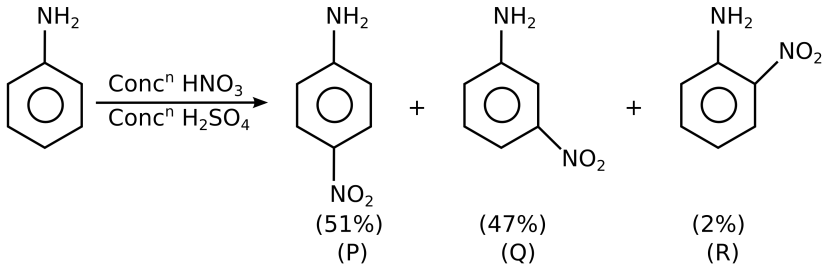
$$E^{\circ}_{Mn^{2+}/Mn} = -1.18V(SRP)$$

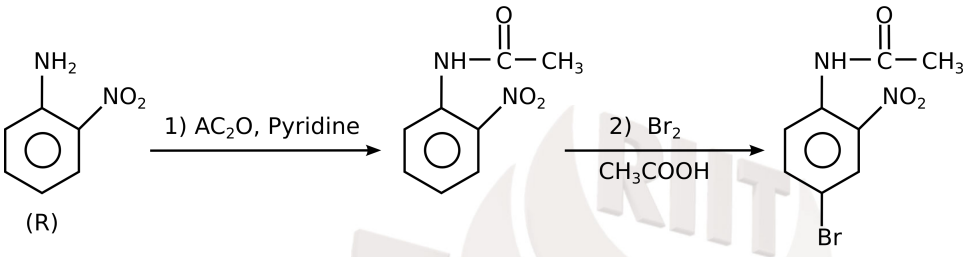
3.

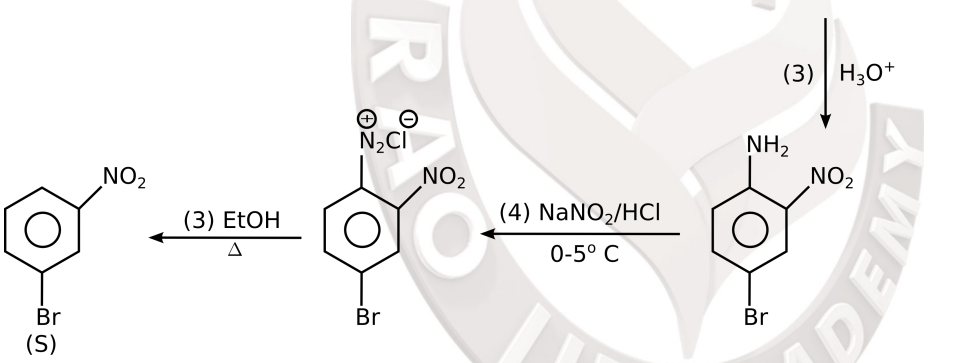
Difficulty : Medium

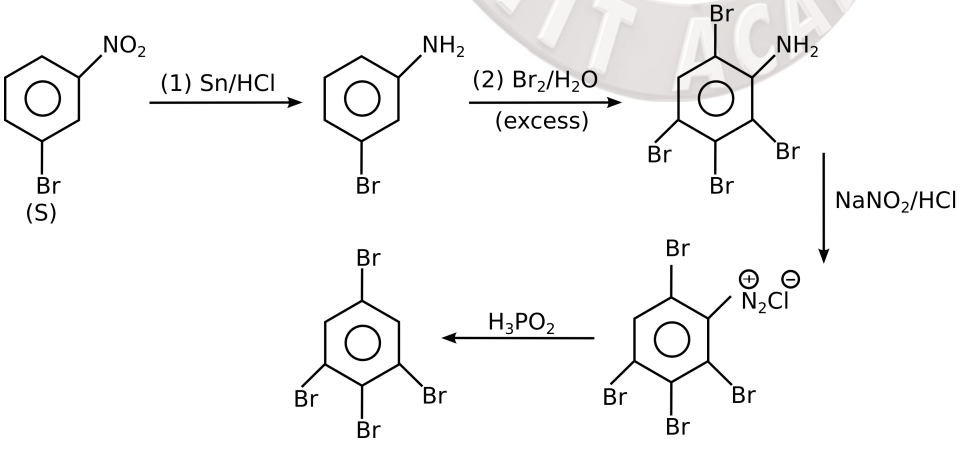
Topics :

Aromatic compounds,







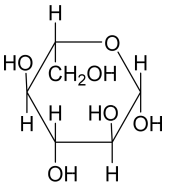


4.

Difficulty : Easy

Topics :

Polymers,
BIOMOLECULES ,



$\beta - L-$ glucopyranose

5.

Difficulty : Medium

Topics :

Chemical Kinetics,

$$A_{(g)} \rightarrow 2B_{(g)} + C_{(g)}$$

$t = 0 \quad P_0 \quad 0 \quad 0$

$t \quad P_0 - x \quad 2x \quad x$

$\therefore P_t = (P_0 - x) + 2x + x$

$$\Rightarrow x = \frac{P_t - P_0}{2}$$

$$\therefore K = \frac{2.303}{t} \log\left(\frac{P_0}{P_0 - x}\right)$$

or $K = \frac{2.303}{t} \log \frac{P_0}{P_0 - \left(\frac{P_t - P_0}{2}\right)}$

$$\Rightarrow K = \frac{2.303}{t} \log\left(\frac{2P_0}{3P_0 - P_t}\right)$$

$$\Rightarrow \log(3P_0 - P_t) = \log 2P_0 - \left(\frac{K}{2.303}\right) t$$

Comparing with $Y = C - mX,$ So it is a straight line with negative slope.

For $t_{1/3}$ _____

$$K = \frac{2.303}{t_{1/3}} \log\left(\frac{P_0}{P_0/3}\right)$$

$\Rightarrow t_{1/3} = \frac{2.303}{K} \log 3 =$ constant

rate constant $'K'$ does not depend on concentration

6.

Difficulty : Difficult

Topics :

Chemical equilibrium,

$\because [A]_{T_2} > [A]_{T_1}$ & $[P]_{T_2} < [P]_{T_1}$

That measn on increasing temperature reaction is shifting in backward direction, so it is exothermic reaction i.e., $\Delta H^\circ < 0$

$\because \frac{\ln K_1}{\ln K_2} > \frac{T_2}{T_1}$

$\Rightarrow T_1 \ \ln K_1 > T_2 \ln K_2$

$\therefore -\Delta G_1^0 > -\Delta G_2^0$

$\Rightarrow -\Delta H^0 + T_1 \Delta S^0 > -\Delta H^0 + T_2 \Delta S^0$

$\Rightarrow T_1 \Delta S^0 > T_2 \Delta S^0$

It is possible when $\Delta S^0 < 0$, because $T_1 < T_2$ (given)

$\because [P] > [A]$....at equilibrium

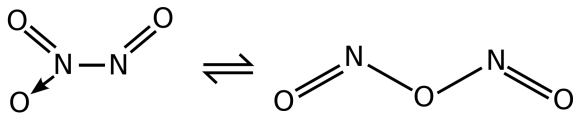
$\Rightarrow K > 1 \Rightarrow \Delta G^0 < 0$

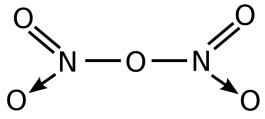
7.

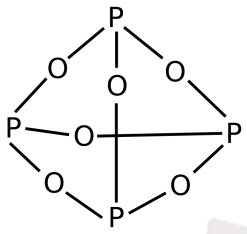
Difficulty : Medium

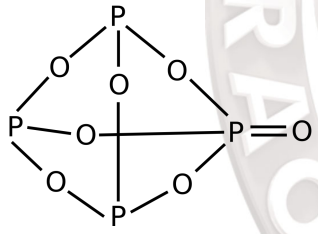
Topics :

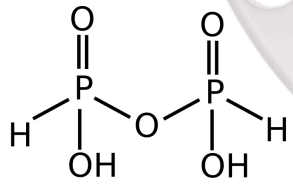
P-block,

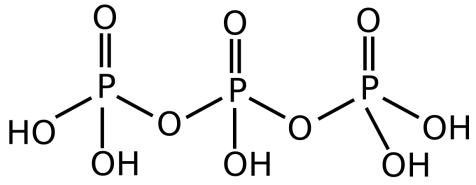
N_2O_3 = 

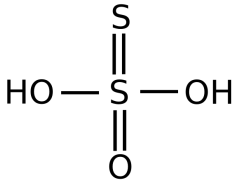
N_2O_5 = 

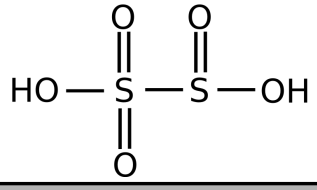
P_4O_6 = 

P_4O_7 = 

$H_4P_2O_5$ = 

$H_5P_3O_{10}$ = 

$H_2S_2O_3$ = 

$H_2S_2O_5$ = 

8.

Difficulty : Difficult

Topics :

Mole concept,
Metallurgy,

$PbS + O_2 \rightarrow Pb + SO_2$

Noumber of equivalents of $O_2 \equiv Pb$

$$\frac{1000}{32} \times 2 = \frac{\text{Mass}}{207} \times 2$$

$$\therefore \ \text{Mass}_{Pb} = \frac{207 \times 1000}{32} gm$$

$$= \frac{207}{32} kg$$

$$= 6.47 \ kg$$

9.

Difficulty : Easy

Topics :

Volumetric analysis,

m.eq of $(COOH)_2 \equiv KM_nO_4$

$$\frac{225}{90} \times 2 = m.mol_{KM_nO_4} \times 5 \quad \text{..... (1)}$$

From given equation _____

$KM_nO_4 \equiv M_nCl_2$

$$m.mol_{KM_nO_4} \times 5 = \frac{\text{Mass}}{126} \times 5 \quad \text{..... (2)}$$

Because $'n'_f$ of KM_nO_4 is 5 in both equations, so

$$\frac{225}{90} \times 2 = \frac{Mass_{MnCl_2}}{126} \times 5$$

$$\therefore \ \boxed{Mass_{MnCl_2} = 126 \ mg}$$

10.

Difficulty : Difficult

Topics :

Stereochemistry,

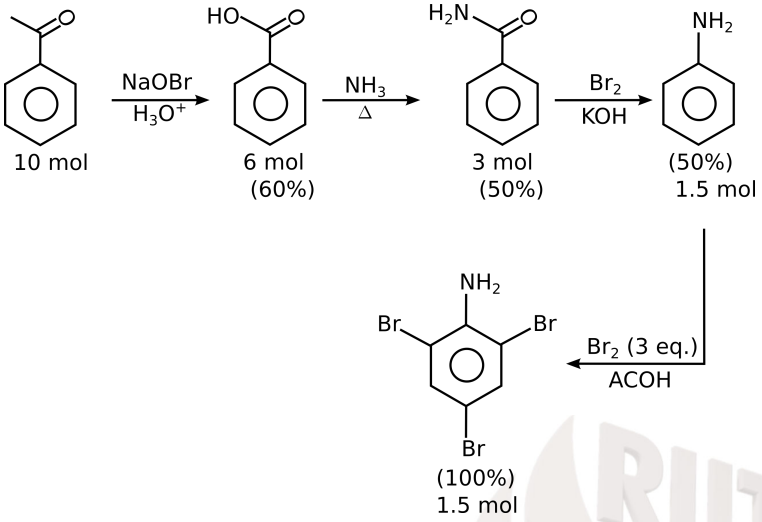
One meso compound and 7 optically active isomers formed.

11.

Difficulty : Medium

Topics :

Aromatic compounds,



$= 1.5 \times 330\ gm$

$= 495\ gm$

12.

Difficulty : Medium

Topics :

Thermodynamics,

$$2Cu_{(s)} + \frac{1}{2}O_{2(g)} \rightarrow Cu_2O_{(s)}, \Delta G^0 = -78,000\ J\ mol^{-1}$$

$$H_2O_{(g)} \rightarrow H_{2(g)} + \frac{1}{2}O_{2(g)}, \Delta G^0 = +1,78,000\ J\ mol^{-1}$$

$$2Cu_{(s)} + H_2O_{(g)} \rightarrow Cu_2O_{(s)} + H_{2(g)}, \Delta G^\circ = 1,00,000\ J\ mol^{-1}$$

To prevent this reaction, $\Delta G \geqslant 0 \Rightarrow \Delta G^\circ \geqslant -RT\ln K$

$\because \Delta G^0 = -RT\ln K$ ..... (considering equality for minimum P_{H2})

$$1,00,000 = -8 \times 1250 \ln\left(\frac{P_{H_2}}{P_{H_2O}}\right)$$

$$1,00,000 = -8 \times 1250\left[\ln\ P_{H_2} - \ln\frac{1\times 1}{100}\right]$$

$$\therefore \ln P_{H_2} = \frac{-1,00,000}{8 \times 1250} - (2 \times 2.3)$$

Minimum $\ln P_{H_2} = -14.6$

13.

Difficulty : Medium

Topics :

Chemical Kinetics,

$$\ln K_f = \ln A_f - \frac{(Ea)_f}{RT} \quad \text{..... (i)}$$

$$\ln K_b = \ln A_b - \frac{(Ea)_b}{RT} \quad \text{..... (ii)}$$

equation (i) - (ii)

$$\ln\left(\frac{K_f}{K_b}\right) = \ln\left(\frac{A_f}{A_b}\right) + \frac{(Ea)_b - (Ea)_f}{RT}$$

$$\ln K_{eq} = \ln 4 + \frac{2RT}{RT}$$

$$= (2 \times 0.7) + 2$$

$$\therefore \ln K_{eq} = 3.4$$

$$\Delta G^0 = -RT \ln K_{eq}$$

$$= -2500 \times 3.4$$

$$= -8500\ J/mol$$

absolute value is $8500\ J/mol$

$$= \boxed{8500}$$

14.

Difficulty : Medium

Topics :

Electrochemistry,

$$[A_{(s)} \rightarrow A^{n+}_{aq.} + ne^-] \times 2$$

$$B^{2n+}_{aq.} + 2ne^- \rightarrow B_{(s)}$$

$$2A_{(s)} + B^{2n+}_{(aq.)} \rightarrow 2A^{n+}_{(aq.)} + B_{(s)}$$

$$\Delta G^0 = -RT \ln K$$

$$= -RT \ln \frac{[A^{n+}]^2}{[B^{2n+}]}$$

$$= -8.3 \times 300 \ln \left(\frac{2^2}{1}\right)$$

$$\Delta G^0 = -8.3 \times 300 \times 0.7 \times 2$$

$$\because \ \Delta G^0 = \Delta H^0 - T\Delta S^0$$

$$\Rightarrow \Delta G^0 = 2\Delta G^0 - T\Delta S^0$$

$$\therefore \Delta G^0 = 2\Delta G^0 - T\Delta S^0$$

$$\therefore \Delta S^0 = \frac{\Delta G^\circ}{T} = \frac{-8.3 \times 300 \times 1.4}{300}$$

$$= -11.62 \ J/K \ mol$$

$$\boxed{-11.62 \ J/K \ mol}$$

15.

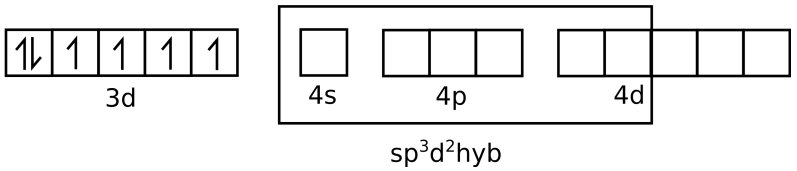
Difficulty : Medium

Topics :

Coordination Compounds,

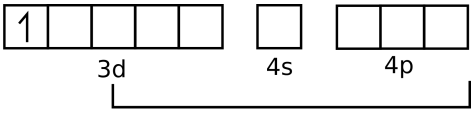
(1) $[Fe\ F]^{4-}, WFL$

Fe^{2+} : [Ar] $3d^6$:



sp^3d^2hyb

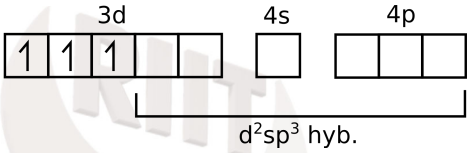
Ti^{3+} = [Ar] $3d^1$:



hyb. d^2sp^3

(2) $[Ti(H_2O)_3Cl_3], WFL$

Cr^{3+} = [Ar] $3d^3$:



d^2sp^3 hyb.

(3) $[Cr(NH_3)_6]^{3+},$

(4) $[FeCl_4]^{2-}, WFL$

Fe^{2+} = [Ar] $3d^6$:

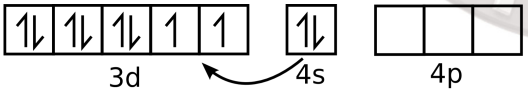
sp^3 hyb.

(5) $[Ni(CO)_4], SFL$

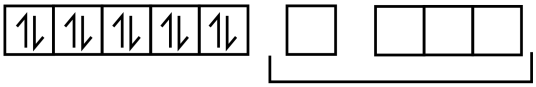
$Ni = [Ar]3d^84s^2$:

due to SFL, back pairing of e^- takes place

Ni :



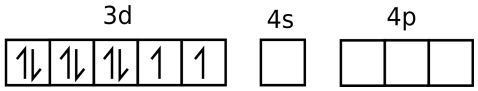
Ni :



sp^3 hyb

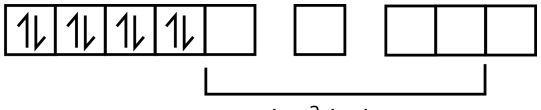
(6) $[Ni(CN)_4]^{2-}, SFL$

Ni^{2+} = [Ar] $3d^8$:



Due to SFL, electron pairing takes place

Ni^{2+} =



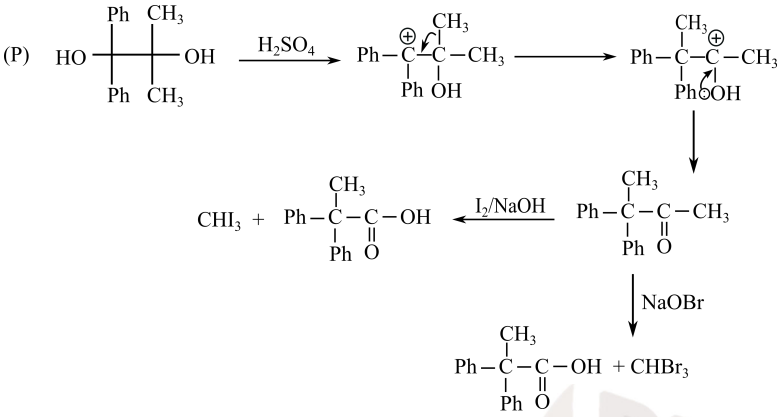
dsp^2 hyb.

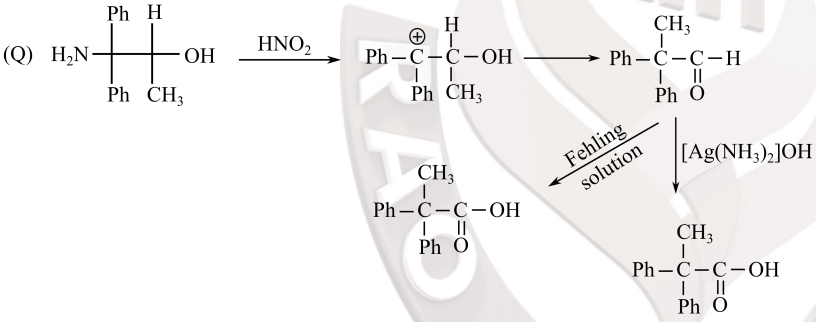
16.

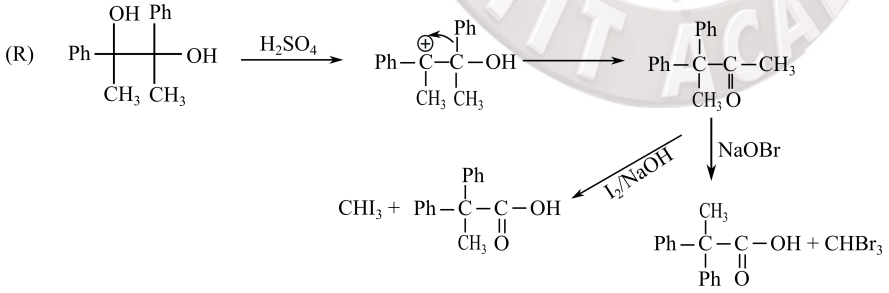
Difficulty : Medium

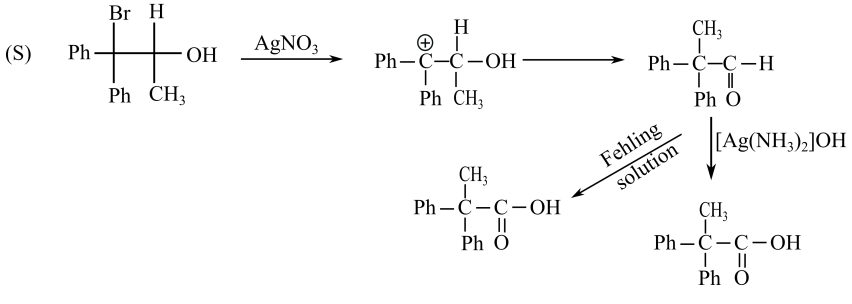
Topics :

Alcohols,
Ethers,







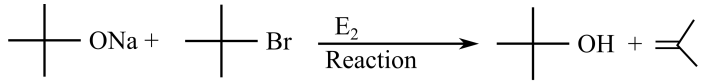


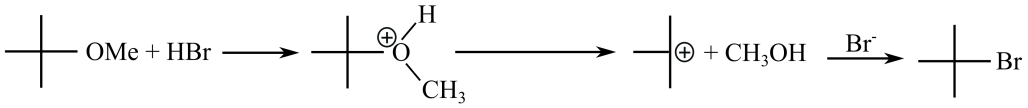
17.

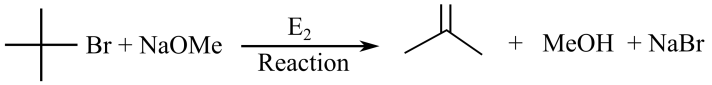
Difficulty : Medium

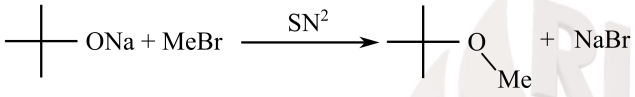
Topics :

Alkyl halides,

(P) 

(Q) 

(R) 

(S) 

18.

Difficulty : Medium

Topics :

Ionic equilibrium,

P. $CH_3COOH + NaOH \rightarrow CH_3COONa + HOH$

$\because$ m. moles $CH_3COOH >$ m. moles $NaOH$ so after reaction, $CH_3COOH \ \& \ CH_3COONa$ will exist as an acidic buffer and its pH or $[H^+]$ is independent of dilution.

$\therefore P \rightarrow 1$

Q. $\because$ m. moles $CH_3COOH =$ m. moles $NaOH$, only CH_3COONa (salt) exists in solution. From salt hydrolysis concept _____

$[H^+] = \sqrt{\frac{K_w.K_a}{C}}$ ..... (for salt of weak acid and strong base)

$\frac{[H^+]_{II}}{[H^+]_I} = \sqrt{\frac{C_I}{C_{II}}} = \sqrt{\frac{C_I}{C_{II/2}}}$

$\therefore [H^+]$ changes to $\sqrt{2}$ times

$Q \rightarrow 5$

R. m. moles $NH_4OH =$ m. moles HCl, only NH_4Cl (salt of w. base and st. acid) will exist in solution

$NH^+ + HOH \rightleftharpoons NH_4OH + H^+$ ..... (from salf hydrolysis)

$\because [H^+] = \sqrt{\frac{C.K_w}{K_b}}$

$\frac{[H^+]_{II}}{[H^+]_I} = \sqrt{\frac{C_{II}}{C_I}}$

$[H^+]_{II} = \frac{[H^+]_I}{\sqrt{2}}$ ..... $(\because C_{II} = \frac{C_I}{2}$

$R \rightarrow 4$

S. Because K_{sp} remains constant, so solubility of salt will not be changed, hence $[OH]^-$ or $[H^+]$ will not be changed after dilution.

$S \rightarrow 1$

MATHEMATICS

1.

Difficulty : Easy

Topics :

Inverse Trigonometric Functions,
Limits,

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{(x+j)-(x+j-1)}{1+(x+j)(x+j-1)}\right)$$

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}(x+j) - \tan^{-1}(x+j-1)$$

$$\therefore\ f_n(x) = \tan^{-1}(x+n) - \tan^{-1} x$$

(A) Zero is not in domain

(B) Zero is not in domain

(C) $\tan f_n(x) = \dfrac{n}{1+(x+n)x}$

$$\therefore \lim_{x\to\infty} \frac{n}{x(x+n)+1} = 0$$ (C is incorrect)

(D) $\lim\limits_{x\to\infty} \sec^2(f_n(x)) = 1 + \lim\limits_{n\to\infty} \tan^2(f_n(x))$

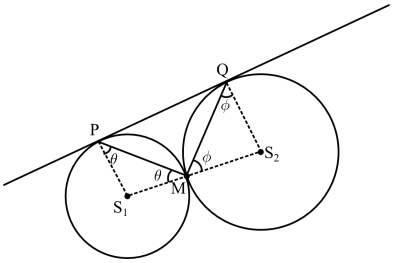
$= 1$ (D is correct)

2.

Difficulty : Medium

Topics :

Circles,



Let $\angle S_1PM = \angle S_1MP = \theta$

and $\angle S_2MQ = \angle S_2QM = \phi$

$\angle MPQ = 90 - \theta$

and $\angle MOP = 90 - \phi$

$\therefore\ \angle PMQ = \phi + \theta$

Also $\phi + \theta + \theta + \phi = 180^\circ$

$\therefore\ \angle PMQ = 90^\circ$

Thus M lies on circle with points P and Q as diameter.

$\therefore$ Locus of M is $E_1 = (x+2)(x-2) + (y-4)(y+5) = 0$

$x \neq -2, 2$

$\left(0, \frac{3}{2}\right)$ does not lie on E_1 (D is correct)

Let midpoint of chord of E_1 be (h, k)

$\therefore\ E_1$ is $x^2 + y^2 - 2y - 39 = 0$

Equation of chord is $T = S_1$

$xh + yk - (y + k) - 39 = h^2 + k^2 - 2k - 39$

Locus of midpoint of such chords passing through $(1, 1)$ is

$h + k - (k + 1) = h^2 + k^2 - 2k$

$\therefore\ E_2 : x^2 + y^2 - x - 2y + 1 = 0$

Although $\left(\frac{4}{5}, \frac{7}{5}\right)$ satisfies the equation, it does not lie on the locus as it is the point corresponding to the point excluded on locus E_1 which is $(-2, 7)$ hence B is correct

$\left(\frac{1}{2}, 1\right)$ does not lie on E_2. $\therefore$ C is incorrect

$\left(0, \frac{3}{2}\right)$ does not lie on E_2 . $\therefore$ D is correct

Also $\because\ x \neq \pm 2$ A is incorrect

3. A,D

Difficulty : Difficult

Topics :

Determinants & Matrices,

$-x+2y+5z=b_1$ ...(i) $2x-4y+3z=b_2$ ...(ii)

$x-2y+2z=b_3$ ...(iii) $\Delta=\begin{vmatrix}-1 & 2 & 5\\ 2 & -4 & 3\\ 1 & -2 & 2\end{vmatrix}=0$

Eq. (ii) + 2 eq (i) gives $13z=b_2+2b_1$

Eq (ii) - 2eq (iii) gives $-z=b_2-2b_3$

$\frac{b_2+2b_1}{13}=2b_3-b_2$ $\therefore\ b_2+2b_1=26b_3-13b_2$

$14b_2+2b_1=26b_3$ $\therefore\ 7b_2+b_1=13b_3$ original condition

(A) $\Delta=\begin{vmatrix}1 & 2 & 3\\ 0 & 4 & 5\\ 1 & 2 & 6\end{vmatrix}\neq 0$ $\therefore$ unique solution irrespective of b_1,b_2,b_3

(B) $x+y+3z=b_1$ ...(i) $5x+2y+6z=b_2$ ...(ii)

$-2x-y-3z=b_3$ ...(iii)

$\Delta=\begin{vmatrix}1 & 1 & 3\\ 5 & 2 & 6\\ -2 & -1 & -3\end{vmatrix}=0$

$\underline{(ii)-2(i)};\underline{(ii)+2(iii)}$ we get

$3x=b_2-2b_1$ $x=b_2+2b_3$

$\therefore$ for infinite solutions $\frac{b_2-2b_1}{3}=b_2+2b_3$

$\therefore\ b_1+b_2+3b_3=0$ which is not the original condition.

$\therefore$ It does not have a solution for all triplets in S.

(C) $-x+2y-5z=b_1$ ...(i) $2x-4y+10z=b_2$ ...(ii)

$x-2y+5z=b_3$ ...(iii)

Clearly planes are parallel $-b_1=\frac{b_2}{2}=b_3$ for infinite solution which need not be true for all values of b in set S, hence this option is not true as well.

(D) $x+2y+5z=b_1$ $2x+3z=b_2$

$x+4y-5z=b_3$

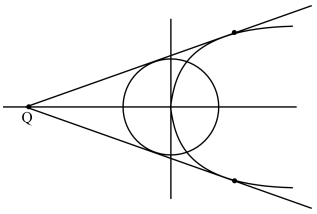
$\Delta=\begin{vmatrix}1 & 2 & 5\\ 2 & 0 & 3\\ 1 & 4 & -5\end{vmatrix}\neq 0$ $\therefore$ unique solution irrespective of b_1,b_2,b_3

4.

Difficulty : Easy

Topics :

Circles,
Ellipse,
Conics,
Area under curve,



$y^2 = 4x$ $\quad x^2 + y^2 = \frac{1}{2}$

Let equation of common tangent be $y = mx + \frac{1}{m}$

$\therefore \left(\frac{1}{m}\right)^2 = \frac{1}{2}m^2 + \frac{1}{2}$ $\quad \therefore \frac{1}{m^2} = \frac{m^2}{2} + \frac{1}{2}$

$m^2(m^2+1) = 2$ $\quad \therefore m^4 + m^2 - 2 = 0$

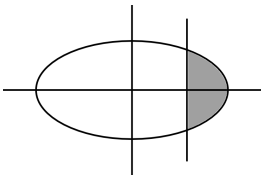
$(m^2+2)(m^2-1) = 0$ $\quad \therefore m = \pm 1$

One of the lines is $y = x + 1$ $\quad Q(-1, 0)$

For ellipse $a = 1, 2b = \sqrt{2}$ $\quad \therefore b = \frac{1}{\sqrt{2}}$

$\therefore$ Equation of ellipse is $\frac{x}{1} + \frac{y^2}{\frac{1}{2}} = 1$ $\quad e^2 = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore e = \frac{1}{\sqrt{2}}$ also $\frac{2b^2}{a} = 1$ (A is correct) For $\frac{x^2}{1} + \frac{y^2}{\frac{1}{2}} = 1$ Area bounded by $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is



Area $= 2\int_{\frac{1}{\sqrt{2}}}^{1} \sqrt{\frac{1-x^2}{2}}\,dx$ $\quad = \sqrt{2}\int_{\frac{1}{\sqrt{2}}}^{1} \sqrt{1-x^2}\,dx$

$x = \sin\theta$ $\quad dx = \cos\theta\, d\theta$

$= \sqrt{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta = \sqrt{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2}\,d\theta$ $\quad = \frac{1}{\sqrt{2}}\left(\theta + \frac{\sin 2\theta}{2}\right)_{\pi/4}^{\pi/2}$

$= \frac{1}{\sqrt{2}}\left[\frac{\pi}{2} - \left(\frac{\pi}{4} + \frac{1}{2}\right)\right] = \frac{1}{\sqrt{2}}\left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi - 2}{4\sqrt{2}}$ $\therefore$ C is correct.

5.

Difficulty : Difficult

Topics :

Complex Numbers,

$sz + t\bar{z} + r = 0$ ...eq (i)

$\bar{s}\bar{z} + \bar{t}z + \bar{r} = 0$ ...eq (ii)

$\bar{s}$ (eq i) $-t$ (eq ii) we get

$s\bar{s}z - t\bar{t}z + r\bar{s} - t\bar{r} = 0$

$\therefore\ z(s\bar{s} - t\bar{t}) = t\bar{r} - r\bar{s}$

(A) If $s\bar{s} \neq t\bar{t}$ i.e. $|s| \neq |t|$. It has unique solution $\therefore$ A is correct

(B) $|s| = |t|$ may have infinite or no solution

(D) If L has more than one solution then no solution and unique solutions are ruled out. $\therefore$ It has infinite solution.

(C) $|z - (1 - i)| = 5$ is a circle.

If L has infinite solution then it lies on the line A line and circle may have atmost two solutions.

6.

Difficulty : Medium

Topics :

Limits,
Application of derivatives,

$$\lim_{t\to x}\frac{f(x)\sin t - f(t)\sin x}{t-x} = \sin^2 x$$

Apply L'Hospital $f(x)\cos x - f'(x)\sin x = \sin^2 x$

$\therefore\ \dfrac{f'(x)\sin x - f(x)\cos x}{\sin^2 x} = -1 \qquad d\left(\dfrac{f(x)}{\sin x}\right) = -1$

$\therefore\ \dfrac{f(x)}{\sin x} = -x + c \qquad \dfrac{f\left(\frac{\pi}{6}\right)}{\frac{1}{2}} = -\dfrac{\pi}{6} + c$

$\therefore\ c = -\dfrac{\pi}{12} + \dfrac{\pi}{12} = 0 \qquad \therefore\ f(x) = -x\sin x$

$\therefore\ f\left(\dfrac{\pi}{4}\right) = -\dfrac{\pi}{4\sqrt{2}}$ A is incorrect.

(B) Cosider $g(x) = -x\sin x - \dfrac{x^4}{6} + x^2$

$$g'(x) = -x\cos x - \sin x - \frac{2}{3}x^3 + 2x$$

$$g''(x) = x\sin x - \cos x - \cos x - 2x^2 + 2$$

$\therefore\ g''(x) = x\sin x - 2x^2 + 2 - 2\cos x$

$$g'''(x) = x\cos x + \sin x - 4x + 2\sin x$$
$$= x\cos x + 3\sin x - 4x$$
$$= x(\cos x - 1) + 3(\sin x - x)$$

$g'''(x) < 0$

$\therefore\ g''(x)$ is decreasing $\qquad \therefore\ g''(x) < g''(0)$

$\therefore\ g''(x) < 0 \qquad \therefore\ g'(x) < g'(0)$

$\therefore\ g'(x) < 0 \qquad \therefore\ g(x)$ is decreasing

$\therefore\ g(x) < g(0) \qquad \therefore\ g(x) < 0$

$\therefore\ -x\sin x - \dfrac{x^4}{6} + x^2 < 0 \qquad \therefore\ f(x) < \dfrac{x^4}{6} - x^2$ B is correct

(C) Apply Rolle's theorem in $[0,\pi]$

$f(0) = f(\pi) = 0$

$\therefore\ f'(\alpha) = 0$ for some $\alpha \in (0,\pi)$ C is correct

(D) $f''(x) = x\sin x - 2\cos x$

$f''\left(\dfrac{\pi}{2}\right) + f\left(\dfrac{\pi}{2}\right) = \dfrac{\pi}{2} - \dfrac{\pi}{2} = 0$ D is correct

7

Difficulty : Easy

Topics :

Definite integration,

$$\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{((x+1)^2(1-x)^6)^{\frac{1}{4}}}dx$$

$$\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{\left[(x+1)^8\left(\frac{1-x}{1+x}\right)^6\right]^{1/4}}dx = 1+\sqrt{3}\int_0^{\frac{1}{2}} \frac{dx}{\left(\frac{1-x}{1+x}\right)^{3/2}(1+x)^2}$$

Put $\frac{1-x}{1+x} = t \qquad \frac{-2}{(1+x)^2}dx = dt$

$$\therefore \; -\frac{(1+\sqrt{3})}{2}\int_0^{1/2} \frac{dt}{t^{3/2}} = \frac{-(1+\sqrt{3})}{2}\left.\frac{t^{-1/2}}{-\frac{1}{2}}\right|_1^{1/3}$$

$$\left.\frac{1+\sqrt{3}}{\sqrt{t}}\right|_1^{1/3} = (1+\sqrt{3})(\sqrt{3}-1) = 2$$

8.

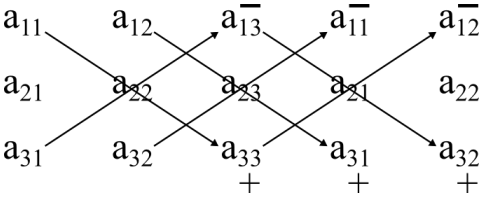
Difficulty : Difficult

Topics :

Definite integration,

$$P = [a_{ij}]_{3\times 3}, \quad a_{ij} \in \{-1, 0, 1\}$$

$$|P| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$



$$= (a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32})$$
$$-(a_{31}a_{22}a_{13} + a_{32}a_{23}a_{11} + a_{33}a_{21}a_{12})$$
$$= (\alpha_1 + \alpha_2 + \alpha_3) - (\beta_1 + \beta_2 + \beta_3)$$

where $\alpha_1 = a_{11}a_{22}a_{33}, \beta_1 = a_{31}a_{22}a_{13}$ etc.

$$\beta_i, \alpha_i = -1, 0, 1 \ \forall \ i = 1, 2, 3$$

for max $|P| \quad \alpha_i = 1, \beta_i = -1 \ \forall \ i = 1, 2, 3$

i.e. $|P| = 6$ but $\alpha_i = 1$ and $\beta_i = -1$ are dependent there is no such combination of a_{ij} s.t. each $\alpha_i = 1$ and each $\beta_i = -1$

Keeping symetrically, 5 anwser is not possible

let us see whether 4 is possible i.e.

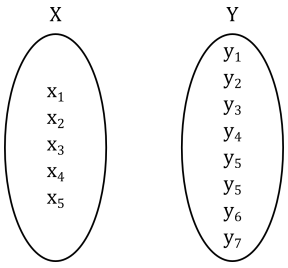
$$(1 + 1 + 0) - (-1 - 1 + 0) \text{ or } \begin{matrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 \end{matrix} \text{ or } (1 + 1 + 1 + 1 + 1 - 1)$$

9. 119

Difficulty : Easy

Topics :

Permutations & Combinations,



$\alpha = {}^{7}C_5 \times 5!$: number of one-one functions from X to Y

β : number of onto functions from Y to X.

We have to distribute 7 items to 5 children such that each gets atleast one item.

Possible cases are

$$\therefore \quad \beta = \frac{7!}{3!(1)^4 4!} \times 5! + \frac{7!}{2!2!2!3!} \times 5!$$

$$\therefore \quad \beta = 140 \times 5!$$

$$\therefore \quad \frac{\beta - \alpha}{5!} = 140 - 21 = 119$$

10.

Difficulty : Easy

Topics :

Differential Equations,

$$\frac{dy}{dx} = (5y+2)(5y-2)$$

$$\int \frac{dy}{(5y+2)(5y-2)} = \int dx$$

$$\int \frac{1}{4}\left[\frac{1}{5y-2} - \frac{1}{5y+2}\right] = x + c$$

$$\frac{1}{4}\ \frac{1}{5}\ \ln\left|\frac{5y-2}{5y+2}\right| = x + c$$

$$f(0) = 0 \qquad \therefore \ \frac{1}{20}\ \ln(1) = c \qquad \therefore \ c = 0$$

$$\therefore \ \ln\left|\frac{5y-2}{5y+2}\right| = 20x$$

$$\text{As} \ \ x \to -\infty. \quad \ln\left|\frac{5y-2}{5y+2}\right| \to -\infty$$

$$\therefore \ \ 5y - 2 \to 0$$

$$\therefore \ \ y \to \frac{2}{5}$$

$$\boxed{\therefore \ \lim_{x \to -\infty} f(x) = 0.4}$$

11.

Difficulty : Easy

Topics :

Functions,
Differential Equations,

$f(x+y) = f(x)f'(y) + f'(x)f(y) \forall x, y \in R$

Put $x = y = 0$

$1 = 2f'(0) \;\; \therefore f'(0) = \frac{1}{2}$

Put $y = 0$

$\frac{f(x)}{2} = f'(x)$

$\therefore 2f'(x) - f(x) = 0$

$\therefore e^{-\frac{x}{2}}(2f'(x) - f(x)) = 0$

$\therefore d(e^{-\frac{x}{2}} f(x)) = 0$

$\therefore e^{-\frac{x}{2}} f(x) = \text{constant} = k$

$\therefore f(x) = ke^{\frac{x}{2}}$

$As \; f(0) = 1 \quad \underline{1 = k}$

$\therefore f(x) = e^{\frac{x}{2}}$

$f(4) = e^2$

$\therefore \boxed{\ln f(4) = 2}$

12.

Difficulty : Easy

Topics :

Three Dimensional Geometry,

Let the image of P ie Q be $(0, 0, \alpha)$

Image of Q about $x + y = 3$ is P

$$\frac{x}{1} = \frac{y}{1} = \frac{z - \alpha}{0} = \frac{(-2)(-3)}{2}$$

$\therefore P(3, 3, \alpha)$

distance of P from x axis $\sqrt{9 + \alpha^2} = 5$

$\therefore \alpha^2 = 16$

$\therefore \alpha = \pm 4$

$\because P$ is in first octant $\alpha = 4$

$\therefore P(3, 3, 4)$

R is image of P about x - y plane.

$R(3, 3, -4)$

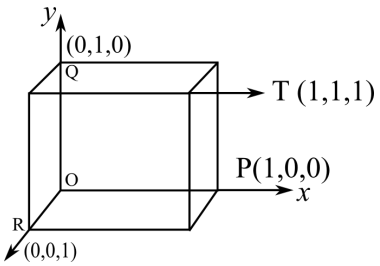
$\therefore l(PR) = 8$

13.

Difficulty : Easy

Topics :

Vectors,



$$S = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

$$\vec{P} = \overrightarrow{SP} = \frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} - \frac{1}{2}\hat{k} = \frac{1}{2}(\hat{i} - \hat{j} - \hat{k})$$

$$\vec{q} = \overrightarrow{SQ} = \frac{1}{2}(-\hat{i} + \hat{j} - \hat{k})$$

$$\vec{r} = \overrightarrow{SR} = \frac{1}{2}(-\hat{i} - \hat{j} + \hat{k})$$

$$\vec{t} = \overrightarrow{ST} = \frac{1}{2}(\hat{i} + \hat{j} + \hat{k})$$

$$= |(\vec{p} \times \vec{q}) \times (\vec{r} \times \vec{t})|$$

$$= |[pqt]\vec{r} - [pqr]\vec{t}|$$

Now $[p\ q\ t] = \frac{1}{8}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} = \frac{1}{2}$

$$[p\ q\ r] = \frac{1}{8}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} = \frac{-1}{2}$$

required value $= \left|\frac{\vec{r} + \vec{t}}{2}\right| = \left|\frac{\vec{k}}{2}\right| = \frac{1}{2}$

14.

Difficulty : Easy

Topics :

Binomial Theorem,

$$\sum_{r=0}^{10} r({}^{10}C_r)^2$$

Consider

$(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1 x + {}^{10}C_2 x^2 \ldots\ldots {}^{10}C_{10} x^{10}$ eq (i)

$(x+1)^{10} = {}^{10}C_0 x^{10} + {}^{10}C_1 x^9 \ldots\ldots {}^{10}C_0$ eq (ii)

Differentiate equation (i) we get

$10(1+x)^9 = {}^{10}C_1 + 2{}^{10}C_2 x + 3{}^{10}C_3 x^2 \ldots\ldots\ldots 10{}^{10}C_{10} x^9$ eq (iii)

Required series is coefficient of x^9 in the product of eq (ii) and eq (iii)

coefficient of x^9 in $10(1+x)^{19}$

$X = 10\ {}^{19}C_{10}$

$X = 10 \times \frac{19}{10}\ {}^{18}C_9$

$X = 19 \times \frac{18}{9}\ {}^{17}C_8 = 19 \times 2 \times\ {}^{17}C_8$

$\frac{X}{1430} = \frac{19 \times 2\ {}^{17}C_8}{1430}$

$= \frac{19 \times 2 \times \frac{17}{8} \times \frac{16}{7} \times \frac{15}{6} \times \frac{14}{5} \times\ {}^{13}C_4}{1430}$

$= 646$

15.

Difficulty : Easy

Topics :

Functions,

$E_1 \frac{x}{x-1} > 0 \quad \therefore E_1 x \in (-\infty, 0) \cup (1, \infty)$

$E_2 \frac{x}{x-1} > 0$

$-1 \leq \ln \frac{x}{x-1} \leq 1$

$\frac{1}{e} \leq \frac{x}{x-1} \leq e$

$\frac{1}{e} \leq 1 - \frac{1}{x} \leq e$

$1 - e \leq \frac{1}{x} \leq \frac{e-1}{e}$

As $1 - e$ is negative

$x \in \left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$

Also $x \in E_1$

$\therefore E_2 \left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$

$f(x) = \log_e \frac{x}{x-1}$

Range of $\log_e \left(\frac{x}{x-1}\right)$ is $R - \{1\}$

$\because \frac{x}{x-1} \neq 1$

$P \to 4$

$g : E_2 \to R$

$g(x) = \sin^{-1} \log_e \left(\frac{x}{x-1}\right)$

Its range contains (0,1) as well as

$Q \to 2$

Also domain of f contains $\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$

$\therefore R \to 1$

domain of g is E_2

$\therefore S \to 1$

16.

Difficulty : Easy

Topics :

Permutations & Combinations,

6 Boys: $M_1, M_2, M_3, M_4, M_5, M_6$

5 Girls : G_1, G_2, G_3, G_4, G_5

(i) Committee of 5 has 3 boys and 2 girls (α_1)

$\therefore \alpha_1 = {}^6C_3 \times {}^5C_2 = 200 \qquad \therefore \alpha_1 = 200$

(ii) Atleast 2 members and equal boys and girls (α_2)

$1G\ 1B \longrightarrow {}^6C_1\ {}^5C_1 \to 30$

$2G\ 2B \longrightarrow {}^6C_2\ {}^5C_2 \to 150$

$3G\ 3B \longrightarrow {}^6C_3\ {}^5C_4 \to 75$

$5G\ 5B \longrightarrow {}^6C_5\ {}^5C_5 \to 6$

$\therefore \boxed{\alpha_2 = 461}$

(iii)

$2G\ 3B \longrightarrow {}^5C_2\ {}^6C_3 \to 200$

$3G\ 2B \longrightarrow {}^5C_3\ {}^6C_2 \to 150$

$4G\ 1B \longrightarrow {}^5C_4\ {}^6C_1 \to 30$

$5G\ 0B \longrightarrow {}^5C_5\ {}^6C_0 \to 1$

$\boxed{\alpha_3 = 381}$

(iv) Total ways of forming comitte of 4 with atleast 2 girls —

Total ways of forming comittee of 4 with atleast 2 girls with $M_1\ \&\ G_1$ both included

$$\begin{pmatrix} {}^5C_2 & {}^6C_2 \\ +{}^5C_3 & {}^6C_1 \\ +{}^5C_4 & {}^6C_6 \end{pmatrix} - \begin{pmatrix} {}^4C_1 & {}^5C_1 \\ +{}^4C_2 & \end{pmatrix}$$

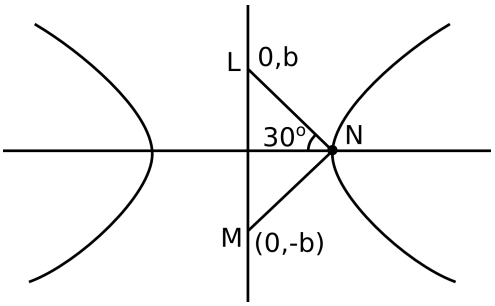
$\boxed{\alpha_4 = 189}$

17.

Difficulty : Easy

Topics :

Hyperbola,
Conics,



$\tan 30^\circ = \frac{b}{a} = \frac{1}{\sqrt{3}}$

$\therefore\ e^2 = 1 + \frac{1}{3} = \frac{4}{3}$

$\therefore e = \frac{2}{\sqrt{3}}$ $\qquad \therefore \boxed{Q \to 3}$

$\frac{1}{2} \times (2b)a = 4\sqrt{3}$

$\therefore\ b(\sqrt{3}b) = 4\sqrt{3}$

$\therefore b = 2$

$\therefore l(C.A) = 4$ $\qquad \therefore \boxed{P \to 4}$

$\therefore a = 2\sqrt{3}$

distance between foci is $2ae : 2(2\sqrt{3})\frac{2}{\sqrt{3}} = 8$

$\boxed{R \to 1}$

$l(R) = \frac{2b^2}{a} = \frac{2(4)}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$ $\qquad \therefore \boxed{S \to 2}$

18

Difficulty : Medium

Topics :

ContinuityDifferentiability & Derivatives,

(i) $f_1(x) = \sin\sqrt{1-e^{-x^2}}$ $\qquad f'(0) = \lim_{h\to 0} \frac{\sin\sqrt{1-e^{-h^2}}}{\sqrt{1-e^{-h^2}}} \frac{\sqrt{1-e^{-h^2}}}{h}$

$= \lim_{h\to 0} \frac{\sin\sqrt{1-e^{-h^2}}}{\sqrt{1-e^{-h^2}}} \frac{\sqrt{1-e^{-h^2}}}{h}$ $\qquad = \lim_{h\to 0} \frac{\sin\sqrt{1-e^{--h^2}}}{\sqrt{1-e^{-h^2}}} \sqrt{\frac{1-e^{-h^2}}{h^2}} \sqrt{h^2}$

$= 1 \times 1 \times \frac{|h|}{h}$

$\therefore LHD \neq RHD$ (both finite) $\qquad \therefore$ continuous but non differentiable

at $x = 0$ $\qquad \boxed{P \to 2}$

(ii) $f_2(x) = \frac{|\sin x|}{\tan^{-1} x} \quad x \neq 0$ $\qquad \lim_{x\to 0} f_2(x) = \frac{|\sin x| \quad |x|}{|x| \ \frac{\tan^{-1}x}{x}} = \frac{|x|}{x}$

Limit does not exist as $LHL \neq RHL$ $\qquad \therefore$ discontinuous at $x = 0$

$\boxed{Q \to 1}$

(iii) $f_3(x) = [\sin\log_e(x+2)]$ $\qquad x\epsilon\left(-1, e^{\frac{\pi}{2}} - 2\right)$

$x + 2 \ \epsilon \left(0, \frac{\pi}{2}\right)$ $\qquad \log_e(x+2)\epsilon\left(0, \frac{\pi}{2}\right)$

$\sin\log_e(x+2)\epsilon(0,1)$ $\qquad \therefore f_3(x) = 0 \ \forall x \ \epsilon \ \left(-1, e^{\frac{\pi}{2}} - 2\right)$

Hence it is differentiable and its derivatives is continuous at $x = 0$

$\boxed{R \to 4}$

(iv) $f_4(x) = x^2 \sin\left(\frac{1}{x}\right); x = 0$

$0 \qquad ; x = 0$

$\lim_{x\to 0} h^2 \sin\left(\frac{1}{h}\right) = 0$ finite $= 0$

$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} h \sin\frac{1}{h} = 0$

$\therefore$ differentiable

Now $f_4'(x) = x^2 \cos\left(\frac{1}{x}\right)\left(\frac{-1}{x^2}\right) + \sin\left(\frac{1}{x}\right) x^2$

$\therefore f_4'(x) = x^2 \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$

Clearly discontinuous at $x = 0$ $\qquad \because \cos\left(\frac{1}{x}\right)$ is oscillatory $\qquad \boxed{S \to 3}$